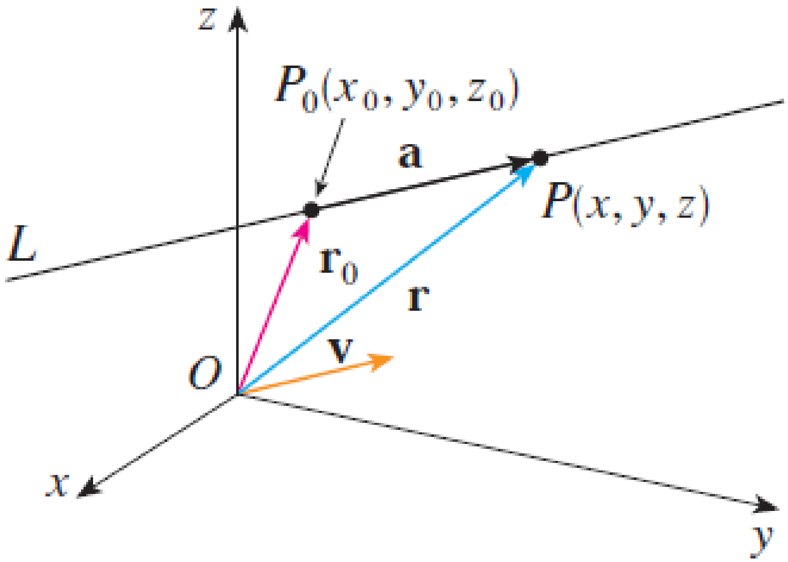
Visual of a Line in 3D



Direction & Position Vectors:

$\mathbf{v} = \langle a, b, c \rangle$ = any vector parallel to the line

$\mathbf{r}_0 = \langle x_0, y_0, z_0 \rangle$ = vector pointing from the origin to some particular point (x_0,y_0,z_0) on the line.

$\mathbf{a} = t\mathbf{v}$ **=** a scale multiple of **v**.

If $\langle x, y, z \rangle = \mathbf{r}_0 + t\mathbf{v}$, then (x,y,z) is a point on the line. We say a **vector form** of the line is:

$$\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$$

Basic Example – Given Two Points:
Find an equation for the line through the points P(1,0,2) and Q(-1, 2, 1).

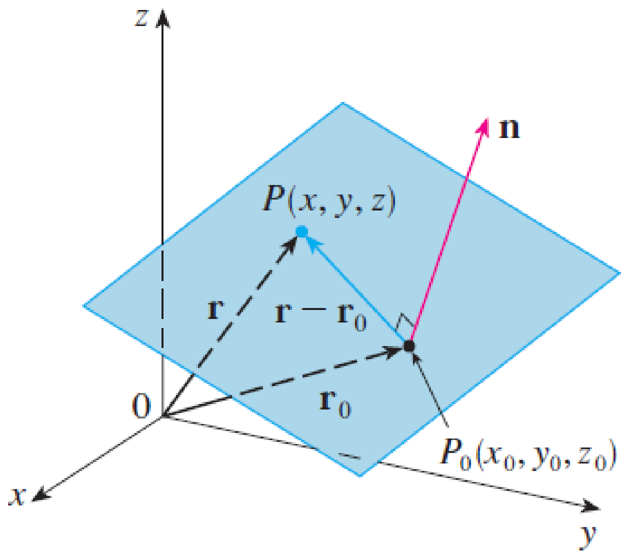
General Line Facts

1. Two lines are **parallel** if their direction vectors are parallel.

2. Two lines **intersect** if they have an (x,y,z) point in common (use different different parameters!)
   Note: The *acute angle of intersection* would be the angle between the direction vectors.

3. Two lines are **skew** if they don’t intersect and aren’t parallel.

Visual of a Plane in 3D



Normal & Position Vectors:

$\mathbf{n}$ **=** $< a, b, c >$ = any vector orthogonal to plane

$\mathbf{r}_0 = < x_0 , y_0, z_0 >$ = vector pointing from the origin to some particular point (x_0,y_0,z_0) on the plane.

Let (x, y, z) be some other point on the plane and consider

$< x - x_0, y - y_0, z - z_0 >$ (denoted by $\mathbf{r} - \mathbf{r_0}$)

Key Observation: $\mathbf{n}$ is orthogonal to $\mathbf{r} - \mathbf{r}_0$.

Thus, we get the **vector form** of the plane:

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$$

Basic Example – Given Three Points:

Find the equation for the plane through the points P(0, 1, 0), Q(3, 1, 4), and R(-1, 0, 0)

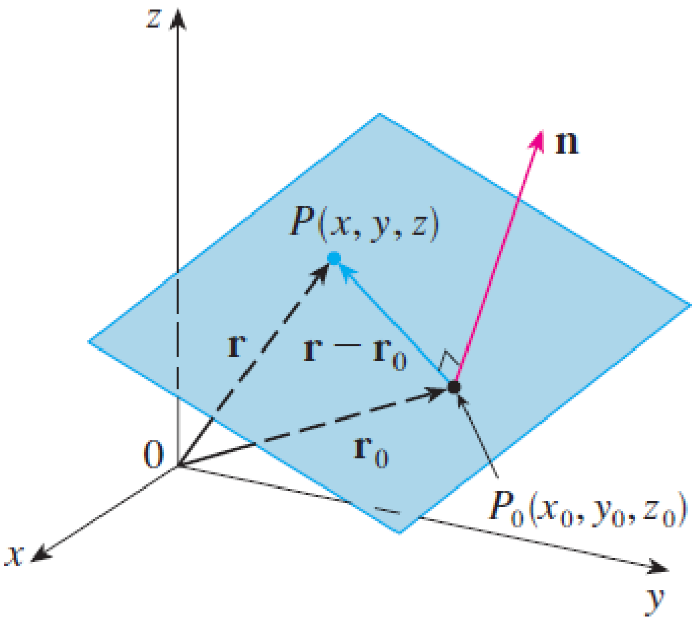
General Plane Facts

1. Two planes are **parallel** if their normal vectors are parallel.

2. If two planes are not parallel, then they must intersect to form a line.

   2a. The *acute angle of intersection* is the angle between their normal vectors.

   2b. The planes are orthogonal if their normal vectors are orthogonal.



Side comment:
If you want the distance between two parallel planes, then
(a) Find any point on the first plane (x_1, y_1, z_1) and any point on the second plane (x_2, y_2, z_2).
(b) Write down the vector

$\mathbf{u} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$

(c) Project **u** onto one of the normal vectors **n**

$|\text{comp}_{\mathbf{n}}(\mathbf{u})|$ = distance between planes